Institut für Analysis

Leibniz Universität Hannover

Workshop on Quantum Harmonic Analysis

August 5-9, 2024

Leibniz Universität Hannover
Welfengarten 1
30167 Hannover

Main Organizers:
Robert Fulsche
Raffael Hagger

Co-Organizers:
Wolfram Bauer
Stine M. Berge
Eske Ewert
Reinhard F. Werner

http://go.lu-h.de/qha

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Program

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Abstracts

QHA and Quantization for the Shearlet Group

Stine Marie Berge

NTNU in Gjøvik

Abstract

In this talk we will delve into quantum harmonic analysis for the shearlet group. In this context, we will define a quantization scheme that works well with the group convolution. If time permits, we will see how this relates to the Wigner distribution for the shearlet group.

Quantum harmonic analysis on spaces of analytic functions

Vishwa Dewage

Clemson University

Abstract

Recently, QHA has been a useful tool to study Toeplitz operators on Fock spaces as seen in the work of Bauer, Fulsche and Hagger. We add to this list a short proof of the Gelfand theory of the radial Toeplitz algebra.

QHA on the Bergman space is more challenging due to the non-commutativity of the group acting on the unit ball. We discuss QHA on the Bergman space. In particular, we modify Suarez's higher order Berezin transform using QHA tools and discuss the long-standing open question of Suarez about approximating an operator S in the Toeplitz algebra by Toeplitz operators with symbols $B_\alpha(S)$.

This talk is based on joint work with Matthew Dawson, Mishko Mitkovski and Gestur Ólafsson.

Scalar products on semi-vector spaces

Carmen Ditscheid

Universität Mannheim

Abstract

Schlather (2024) suggests an underlying algebraic structure for entropies and derives a generic real-valued scalar product. While extending the theory to semi-vector spaces, we introduce a complex-valued scalar product. The extension portrays different axes and their working together. The formulae seem to remind some of the mathematical structures in quantum mechanics.

Reference: Schlather, M. (2024) An algebraic generalization of the entropy and its application to statistics.
https://arxiv.org/pdf/2404.05854

QHA via the Banach Gelfand Triple II: Structure preserving finite dimensional approximations

Hans Georg Feichtinger

University of Vienna

Abstract

In some sense this talk builds on the first ideas communicated at the workshop in Trondheim in June 2023, indicating that the Banach Gelfand Triple (S_0, L^2, S_0^*), consisting of Feichtinger's algebra, the Hilbert space L^2 and the dual space S_0^* of "mild distributions", provide a convenient platform for the treatment of questions arising in Quantum Harmonic Analysis. This Banach Gelfand Triple is invariant under the Fourier transform, but there is also a Kernel Theorem and a Kohn-Nirenberg calculus. Convolution of operators and functions, operator Fourier transforms, and so on make sense in this context.

Since the space of mild distributions also contains discrete and periodic signals one can interpret functions on the cyclic group (using the FFT) alternatively as elements of S_0^*. The fact that any (continuous) function f in S_0 can be approximately recovered from a coarsly periodized and sufficiently fine sampled version of f indicates that there is a way to approximate continuous, non-periodic objects by finite dimensional objects. Similar considerations hold true for operators and allow in this way to provide numerical (FFT-based) algorithms for the approximate (constructive realizable) computations of they key operators in QHA.

QHA and limit operators: A good combination

Robert Fulsche

Leibniz Universität Hannover

Abstract

In this talk, we will try to convince the audience that combining the theory of quantum harmonic analysis with the theory of limit operators is a good and fruitful idea. We start by properly introducing the concept of limit operators. We present the basic structure result, which shows that (families of) limit operators appear naturally when working with the algebra $\mathcal{C}_1$, which is a natural framework in QHA. Finally, we will show how limit operator theory can be used to obtain results in QHA, namely we discuss Fredholm theory, the algebra theorem in correspondence theory, as well as recent results on Tauberian theorems for operators.

This talk is based on joint works with R. Hagger, F. Luef and R. F. Werner.

Wigner analysis of operators

Gianluca Giacchi

University of Bologna

Abstract

The original idea of combining operator kernels with the Wigner distribution, pioneered by E. Wigner in 1932, laid the groundwork for a novel approach in operator calculus. While Wigner's initial formulation provided a conceptual framework, its refinement and rigorous formulation have been recently achieved by E. Cordero, G. Giacchi and L. Rodino. We explore the utilization of Wigner kernels to derive robust representation formulas for Schrödinger propagators. This approach facilitates the definition of algebras of Fourier integral operators and provides a novel framework for representing Gabor matrices. We offer a comprehensive overview of these advancements, highlighting their mathematical significance and potential applications.

On Toeplitz Quantum States

Maurice de Gosson

University of Vienna

Abstract

Density operators are positive semidefinite operators with trace one representing the mixed states of quantum mechanics. The purpose of this contribution is to define and study a subclass of density operators on $L^2(\mathbb{R}^n)$, which we call Toeplitz density operators. They correspond to quantum states obtained from a fixed function ("window") by position-momentum translations and reduce in the simplest case to the anti-Wick operators considered long ago by Berezin and extensively studied by Cordero and others. The rigorous study of Toeplitz operators requires the use of classes of functional spaces defined by Feichtinger.

Operator algebras on locally compact abelian groups

Raffael Hagger

Christian-Albrechts-Universität zu Kiel

Abstract

Let G be a locally compact abelian group and $\widehat{G}$ its Pontryagin dual. For $(g,\xi) \in G \times \widehat{G}$ we define unitary operators $U_{(g,\xi)} : L^2(G) \to L^2(G)$ by

$$U_{(g,\xi)} f(x) = \varphi_{g,\xi}(x) f(g^{-1}x),$$

where $\varphi_{g,\xi}$ is a unimodular factor satisfying some cocycle condition to ensure that $(g,\xi) \mapsto U_{(g,\xi)}$ forms an irreducible projective representation of $G \times \widehat{G}$. Consider the C^*-algebra

$$\mathcal{C}_1(G) := \left\{ T \in \mathcal{L}(L^2(G)) : (g,\xi) \mapsto U_{(g,\xi)} T U_{(g,\xi)}^{-1} \text{ is } \|\cdot\|\text{-continuous} \right\}.$$

By Werner's correspondence theorem, this C^*-algebra corresponds to $\mathrm{BUC}(G)$ and is therefore one of the natural algebras to study. Some characterizations of this algebra within QHA are somewhat expected. But more surprisingly, $\mathcal{C}_1(G)$ is also equal to an algebra of band-dominated operators, the natural algebra to consider in limit operator theory. This observation leads to some interesting new insights as it allows to combine tools from QHA and limit operator theory to characterize spectral properties such as compactness and Fredholmness of operators on $L^2(G)$.

Based on joint work with Robert Fulsche.

Measure-operator convolutions and applications to mixed-state Gabor multipliers

Simon Halvdansson

NTNU Trondheim

Abstract

One of the core operations of quantum harmonic analysis is convolutions between functions and operators. In this talk, we show how recent results by Feichtinger can be used to extend the standard definition to include convolutions between measures and operators. Many properties of function-operator convolutions carry over to this setting and allow us to prove novel results on the distribution of eigenvalues of mixed-state Gabor multipliers and derive a version of the Berezin-Lieb inequality for lattices. New results on the continuity of Gabor multipliers with respect to lattice parameters, masks and windows as well as their ability to approximate localization operators are also derived using this framework.

Based on joint work with Hans Feichtinger and Franz Luef.

Reflection positivity

Palle Jorgensen

University of Iowa

Abstract

The talk aims to extend tools from reflection positivity (RP) to non-commutative harmonic analysis, spectral theory, and new duality theory for unitary representations of Lie groups. In its original form reflection positivity (RP) has come to serve as a crucial link between problems in quantum physics and in mathematics. More precisely, the original variant of RP serves to link Euclidean field theory (math) to relativistic quantum field theory (physics). The basic symmetry groups of the two sides are different, e.g., the Poincare group vs the Euclidian group. Hence the associated harmonic analysis and spectral theory are different of course. In recent work by the speaker and multiple co-authors have aimed to expand the framework of RP to the theory of representations of Lie group, spectral theory, and operator algebras. Among other things, this viewpoint yields insight into the role of the Markov property, as opposed to Osterwalder-Schrader positivity. Recently the general principle of Reflection positivity (RF) has proved useful in diverse areas of mathematics and in many neighboring areas. It combines powerful tools from analysis, from geometry, from representation theory to questions in quantum physics. For example, due to work by many authors, the RF-correspondence has served to link abelian (commutative) properties of Gaussian processes/fields in the Euclidean setting, to the context of non-commutativity in the study of quantum fields. And by now, RP has further become a powerful tool in non-commutative harmonic analysis, and in the theory of unitary representations of Lie groups.

Joint measurement of quasi-free observables in phase space

Jukka Kiukas

Aberystwyth University

Abstract

Existence of observables which cannot be jointly measured is a fundamental feature of quantum theory. Observables represented by self-adjoint operators are jointly measurable exactly when they commute, and phase space quantum mechanics is naturally based on the non-commutativity of the canonical position-momentum pair. However, introducing noise to such observables by convolving them with probability measures leads to observables represented by general positive operator valued measures, in which case commutativity is no longer necessary for joint measurability, and the existence of joint observables becomes an interesting problem.

It turns out that joint measurability problems of this type can be conveniently described within the general framework of quasi-free observables, which are defined by covariance with respect to suitably restricted phase space translations. Interestingly, by applying recent results by L. Dammeier and R.F. Werner on quantum-classical hybrid systems, one can interpret each such joint measurability structure in terms of canonical coordinate observables measured on a single state of an auxiliary hybrid system which includes classical degrees of freedom. Here I present this general formulation and discuss in detail some relevant particular cases, such as sets of noisy quadrature observables.

From QHA to QTFA

Franz Luef

NTNU Trondheim

Abstract

In this talk I describe the transition from quantum harmonic analysis to quantum time-frequency analysis, which is motivated by boundedness results for pseudodifferential operators on modulation spaces developed by Cordero, Groechenig and Heil. The polarized Cohen's class turns out to play the role of the STFT of a function and we explore this aspect of QTFA in detail. This is joint work with Henry McNulty (Cognite AS, NTNU Trondheim).

Quantum Time–Frequency Analysis and Pseudodifferential Operators

Henry McNulty

NTNU Trondheim

Abstract

We introduce Quantum Time-Frequency Analysis, which expands the approach of Quantum Harmonic Analysis to include modulations of operators in addition to translations. This is done by a projective representation of double-phase space, and we consider the associated matrix coefficients and integrated representation. This leads to the polarised Cohen's class, which is an isomorphism from Hilbert-Schmidt operators to a reproducing kernel Hilbert space, and has orthogonality relations similar to many objects in classical time-frequency analysis. By considering a class of windows for the polarised Cohen's class that is smaller than the class of Hilbert-Schmidt operators, then we find spaces of modulation spaces of operators, and we consider the properties of these spaces, including mapping properties between function modulation spaces. Using the tools of time-frequency analysis we examine different discretisations and approximations of operators in operator modulation spaces.

Analytic continuation of Bargman spaces and Toeplitz operators

Gestur Ólafsson

Louisiana State University

Abstract

Bergman spaces were originally defined as the reproducing kernel Hilbert space $\mathbb{A}^2_\sigma(\mathbb{D})$ of holomorphic functions on a bounded domain $\mathbb{D} = G/K$ which are also square integrable with respect to certain probability measure μ_σ, $\sigma > c$, c a constant depending on $\mathbb{D}$. A Toeplitz operator with symbol $\varphi \in L^\infty(\mathbb{D})$ is the operator $\mathrm{T}^\sigma_\varphi(F) = \mathrm{pr}(\varphi F)$ where $\mathrm{pr} : L^2(\mathbb{D}, \mu_\sigma) \to \mathbb{A}^2_\sigma(\mathbb{D})$ is the orthogonal projection. It was shown shown in 1976 by Rosse and Vergne that one can extend the definition of the Bergman spaces to a bigger set of parameters. Later in 2015 Bommier-Hato, Englis and Youssfi showed that the Toeplitz operators $(\mathrm{T}^\sigma_\varphi)_{\sigma>c}$ also have analytic continuation at least for symbols that are smooth enough.

We will discuss the the basic theory in general but then concentrate on examples, mostly the ball $\mathrm{B}^n \subset \mathbb{C}^n$ where the Bergman spaces are first defined for $\sigma > n$, but then the definition is extended to all $\sigma > 0$. It can be shown for $n \geq \sigma > 0$ there is no probability measure on the ball such that the elements in the Bergman space are square integrable with respect to that measure. Hence the standard definition of Toeplitz operators does not work.

After introducing the analytic continuation in two different ways we will discuss Toeplitz operators for the extended set of parameters. Finally we discuss how invariance under action of maximal abelian subgroups of $\mathrm{SU}(1, n)$, well known for $\sigma > n$, lead also to abelian C^* algebras in the extended situation and connect this to previous work with Dawson and Quiroga-Barranco. Joint work with Khalid Bdarneh.

Decoupling for Schatten class operators in the setting of Quantum Harmonic Analysis

Helge Jørgen Samuelsen

NTNU Trondheim

Abstract

Given $\Omega \subseteq \mathbb{R}^{2d}$ and a partition $\mathcal{P}_\Omega$ of Ω, we define the decoupling constant $\mathcal{D}^{\mathscr{C}}_{p,q}(\mathcal{P}_\Omega)$ as the smallest constant for which the inequality

$$\left\| \sum_{\theta \in \mathcal{P}_\Omega} f_\theta \right\|_{L^p(\mathbb{R}^{2d})} \leq \mathcal{D}^{\mathscr{C}}_{p,q}(\mathcal{P}_\Omega) \left(\sum_{\theta \in \mathcal{P}_\Omega} \|f_\theta\|^q_{L^p(\mathbb{R}^{2d})} \right)^{\frac{1}{q}},$$

holds for any collection of functions f_θ where the symplectic Fourier transform of f_θ is supported on θ. We refer to this as decoupling for the set Ω, and the main goal is to control the size of the decoupling constant $\mathcal{D}^{\mathscr{C}}_{p,q}(\mathcal{P}_\Omega)$.

Decoupling has had an enormous impact in different areas of analysis in recent times. First introduced by Wolff in order to study the wave equation, it later grew to have applications in harmonic analysis, number theory and the theory of partial differential equations. It was also the crucial component in Bourgain, Demeter and Guth's proof of the Vinogradov's mean value conjecture in 2016.

In this talk, we extend the notion of decoupling to bounded linear operators on $L^2(\mathbb{R}^d)$. We introduce the quantum decoupling constant $\mathcal{D}^{\mathscr{Q}}_{p,q}(\mathcal{P}_\Omega)$, and prove an equivalence relation between the classical and quantum decoupling constant for bounded sets.

Moduli of continuity of spectra of a class of pseudodifferential operators and applications to Gabor frame bounds

Michael Speckbacher

Acoustics Research Institute Vienna

Abstract

We study one-parameter families of pseudodifferential operators whose Weyl symbols are obtained by dilation and a smooth deformation of a symbol in a weighted Sjöstrand class. We show that their spectral edges are Hölder continuous functions of the dilation or deformation parameter. Suitably local estimates hold also for the edges of every spectral gap. Our argument first extends Bellissard's seminal results on the Lipschitz continuity of spectral edges for families of operators with periodic symbols to a large class of symbols with only mild regularity assumptions and subsequently uses an approximation result to obtain Hölder-continuity estimates. The abstract results are then applied to families of Gabor frame operators where the nonuniform set of time-frequency shifts is dilated by a parameter α. This settles a question about the precise blow-up rate of the condition number of Gabor frames near the critical density.

Hagedorn states and the localization problem for Cohen's class

Erling Svela

NTNU Trondheim

Abstract

Let $\Omega \subset \mathbb{R}^{2d}$, and let S be a positive trace class operator on $L^2(\mathbb{R}^d)$. The localization problem for Cohen's class consists of finding an $f \in L^2(\mathbb{R}^d)$ that maximizes the quantity

$$\frac{\int_\Omega \check{S} \star (f \otimes f)(z)\, dz}{\|f\|_2},$$

where $\check{S} \star (f \otimes f)$ is the operator-operator convolution between $\check{S}$ and the rank-one operator $f \otimes f$. Inspired by methods from time-frequency analysis we examine properties of the solutions of the localization problem, both in the atomic and general case. Based on a generalization of Daubechies' theorem for localization operators we also present an explicit solution to the localization problem in the case where Ω is a Reinhardt domain and S is a mixed Hagedorn state.

Modulation spaces, harmonic analysis and pseudo-differential operators

Joachim Toft

Linnæus University

Abstract

In the present talk we present recent results on composition, continuity and Schatten-von Neumann (SvN) properties for operators and pseudo-differential operators (ΨDOs) when acting on modulation spaces. For example we present necessary and sufficient conditions in order for the Weyl product should be continuous on modulation spaces. Such question is strongly connected to questions whether compositions of ΨDOs with symbols in modulation spaces remain as ΨDOs with a symbol in a modulation space.

We also present necessary and sufficient conditions for ΨDOs with symbols in modulation spaces should be SvN operators of certain degree in the interval $(0, \infty]$. Note here that there are so far only few results in the literature on SvN operators with degrees less than one.

Parts of the talk are based on joint works with D. Bhimani, Y. Chen, E. Cordero, R. Manna and P. Wahlberg.

Conference dinner

The dinner will take place in the restaurant *Meiers Lebenslust* (Osterstraße 64, www.meiers-lebenslust.de) at 18:30. It is located in the center of Hannover. You can go there:

By foot: After leaving the university building through the main entrance, turn left and follow the main street for approximately 2.5 kilometers.

By public transport: From the tram stop right in front of the university building, take tram lines 4 (direction "Roderbruch") or 5 (direction "Anderten") and leave at the stop "Aegidientorplatz", which is 4 stations away and takes about 5 minutes. From there, it is a walk of about 200 meters to the restaurant.

By taxi: If you want to take a taxi to the restaurant, please get in touch with the organizers. We will gladly organize a taxi for you.

A short history of Hannover and the Welfenschloss

The city of Hannover (traditionally spelled as "Hanover" in English) goes back to a medieval founding, which already existed in the 11th century, and the name of the city most likely originates from those times: The initial settlement was on the shore of the river *Leine*, yet high enough for being protected from the yearly floodings. It was lying on the *Honovere* (medieval German for *hohes Ufer* - high shore). Since the settlement was located next to an important bridge over the *Leine* where two important trade routes met, it soon developed to be an important trade center. Yet, for a long time its importance was overshadowed by nearby flourishing cities such as Hamburg, Bremen or Brunswick.

Reign over the area in which Hannover is located changed several times. Roughly speaking, from the mid 13th century Hannover belonged to the Principality of Calenberg (Fürstentum Calenberg), which was part of the Duchy of Brunswick-Lüneburg (Herzogtum Braunschweig-Lüneburg) and therefore under the rule of a branch of the House of Welf. During the Thirty Years' War, in the 17th century, Georg of Calenberg made Hannover the capital of the Principality. In 1692, the Principality of Calenberg, being the most powerful of the states from the Duchy of Brunswick-Lüneburg, was elevated to the Electorate of Brunswick-Lüneburg (Kurfürstentum Braunschweig-Lüneburg) and the Prince of Calenberg became one of the nine Prince-electors (Kurfürsten) of the Holy Roman Empire. Later on, the Electorate of Brunswick-Lüneburg was colloquially known as the Electorate of Hannover.

After the death of Queen Anne of Great Britain and Ireland in 1714, the Prince-electors of Hannover also became Kings of Great Britain and Ireland, ruling over both states. Following the Napoleonic Wars, it was decided in the Congress of Vienna in 1814 that the Electorate of Brunswick-Lüneburg would become a kingdom, now officially called the Kingdom of Hannover. In 1837, the personal union between the

Historical picture of the Welfenschloss from around 1895.
Source: de.wikipedia.org/wiki/Welfenschloss

King of Great Britain and the King of Hannover ended: Victoria became Queen of Great Britain and Ireland. Due to different rules organizing the succession in Hannover, which preferred male successors, Ernst August, an uncle of Victoria, became King of Hannover. The son of Ernst August, King Georg V. of Hannover, later ordered the construction of a new residency in Hannover, the *Welfenschloss* (castle of the House of Welf), which was built between 1857 and 1866. In 1866, shortly before the construction of the *Welfenschloss* was completed, Prussia annexed Hannover during the Austro-Prussian War, turning it into the Prussian Province of Hannover. The King of Hannover had to resign and the Welfenschloss was never used as a residence. In 1879, it was given to the Technische Hochschule Hannover (now: Leibniz Universität Hannover).

Hannover formally kept its status as a Prussian province until 1946. Upon the founding of the federal state of Lower Saxony (Niedersachen) in that year, Hannover was chosen as its capital. The horse statue in front of the Welfenschloss (the *Niedersachsenross*), which you can see upon leaving the building through the main entrance, is part of the coat of arms of Lower Saxony.